–3 Å flux was below the sensitivity threshold of the experiment on OSO-5. During the only other periods when the background 0.5–3 Å flux was found to be above the threshold level, June 1–June 17, 1969, November 17–30, 1969, and February 1–4, 1970, electron temperatures were found to be in the seven–eight million degree range during periods of flare activity.

The 0.5–3 Å experiment designed for SOLRAD 10, which is to be launched in 1971, will be approximately six times more sensitive than the OSO-5 experiment. Hopefully, it will significantly increase the periods when background 0.5–3 Å flux is measurable and enable a better assessment of the variation of nonflare electron temperatures with flare activity to be made.

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Hypersonic Limits for Laminar, Constant–Pressure Boundary Layers

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Nomenclature

$(c_f)_e;(c_f)_r$	$= \tau_w/\frac{1}{2}\rho_e u_e^2; = \tau_w/\frac{1}{2}\rho_r u_e^2$, friction coefficients
c_p	= specific heat
k_{eq}	= equilibrium thermal conductivity
M	= Mach number
n	= 0 for planar flow, = 1 for axisymmetric flow
p	= static pressure
Pr_{eq}	$= \mu c_p/k_{eq}$
q	= heat flux
$Re_e;Re_r$	$= \rho_e u_e x/\mu_e; = \rho_r u_e x/\mu_r$, Reynolds numbers
$st_e;st_r$	$= q_w/\frac{1}{2}\rho_e u_e^3; = q_w/\frac{1}{2}\rho_r u_e^3$, Stanton numbers
u,v	= velocity components parallel and normal to body
x,y	= coordinates parallel and normal to body
γ	= ratio of specific heats
μ	= viscosity
ρ	= density
τ	= shear stress

Subscripts

e	= boundary layer edge value
r	= reference state
w	= wall value

Introduction

IN this note the laminar boundary layer problem is formulated in such a way that the skin friction parameter and Stanton number parameter approach constant values for Mach numbers much greater than one. The results obtained provide a rapid method of estimating skin friction and heat transfer for $M_e > 10$.

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Analysis

In the low speed limit $M_e \rightarrow 0$, it is well known that the friction and heat-transfer parameters for laminar flow over an insulated flat plate approach constant values. This result is emphasized in Fig. 1a by plotting the von Kármán and Tsien[1] result for $\mu \propto T^{0.76}$, $Pr = 1.0$ on semi-logarithmic coordinates. As M_e increases, one notes that $(c_f)_e Re_e^{1/2}$ and $st_e Re^{1/2}$ decrease continually. However, in the high-speed limit $M_e \gg 1$ (or $M_e^{-1} \rightarrow 0$) it may be possible to obtain "hypersonic limits" for these parameters. Such an analysis is given below for laminar, constant pressure (i.e., flat plates, wedges, cones) boundary layers.

The boundary layer equations for constant pressure

$$(\rho u r^n)_x + (\rho v r^n)_y = 0 \tag{1}$$

$$\rho u u_x + \rho v u_y = (\mu u_y)_y \tag{2}$$

$$\rho u h_x + \rho v h_y = [(k_{eq}/c_p)h_y]_y + \mu u_y^2 \tag{3}$$

are transformed by defining the coordinate transformations

$$\eta \equiv \frac{u_e r^n}{(2\zeta)^{1/2}} \int_0^y \rho dy^*, \quad \zeta \equiv \int_0^x \rho_r \mu_r u_e r^{2n} dx^* \tag{4}$$

and a nondimensional stream function

$$f \equiv \psi/(2\zeta)^{1/2} \tag{5}$$

Equation (4) is identical to the Levy-Lees transformation except for the introduction of "reference values" of ρ and μ defined by

$$\rho_r = \rho(h_r,p_e) \qquad \mu_r = \mu(h_r,p_e) \tag{6}$$

where the reference enthalpy $h_r = \frac{1}{2}u_e^2$. After defining the nondimensional variables

$$\tilde{u} = u/u_e, \quad \tilde{h} = h/h_r, \quad \tilde{\rho} = \rho/\rho_r, \quad \tilde{\mu} = \mu/\mu_r \tag{7}$$

and substituting Eqs. (4–7) into Eqs. (1–3), the boundary-layer equations may be written as

$$[\tilde{\rho}\tilde{\mu}\tilde{u}']' + f\tilde{u}' = 0 \tag{8}$$

$$[(\tilde{\rho}\tilde{\mu}/Pr_{eq})\tilde{h}']' + f\tilde{h}' + 2\tilde{\rho}\tilde{\mu}(\tilde{u}')^2 = 0 \tag{9}$$

where

$$f = \int_0^\eta \tilde{u} d\eta^*$$

and $(\ )'$ indicates differentiation with respect to η. It should be noted that Eq. (9) has no explicit Mach number dependence. The boundary conditions (for a constant temperature wall) are

$$\begin{aligned} &\text{at } \eta = 0; \; f = 0, \; \tilde{u} = 0, \; \tilde{h} = \tilde{h}_w \\ &\text{at } \eta = \infty; \qquad \tilde{u} = 1, \tilde{h} = \tilde{h}_e \end{aligned} \tag{10}$$

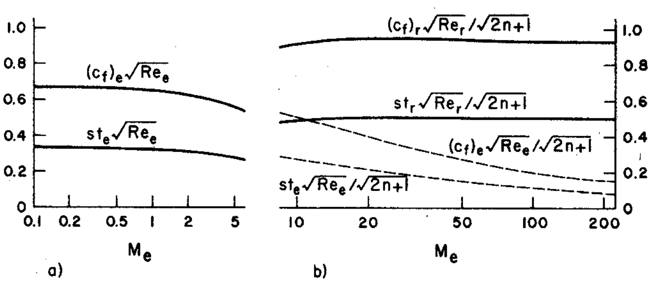

Fig. 1 Low Mach number and high Mach number limits for the friction and heat transfer parameters.

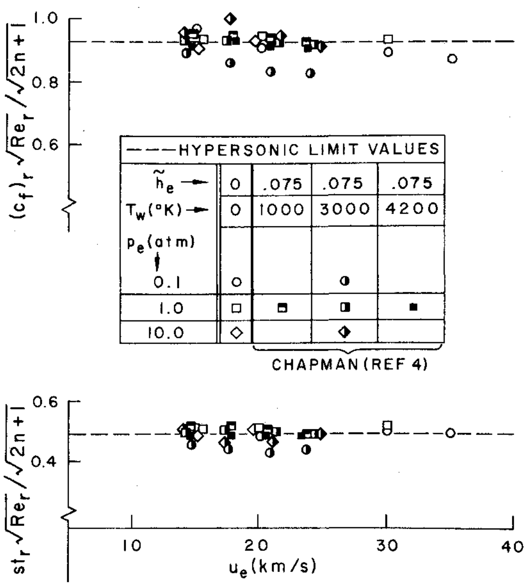

Fig. 2 Comparison of approximate and exact solutions for the friction and heat transfer parameters.

In the hypersonic limit, $\tilde{h}_w \rightarrow 0$ for a finite wall temperature and $\tilde{h}_e \rightarrow 0$ for a flat plate.†

Equations (7–10) have been solved for $\tilde{h}_e = \tilde{h}_w$ using approximations to the gas properties of the form

$$\tilde{\mu} = \tilde{h}^{\alpha}, \quad \tilde{\rho} = \tilde{h}^{-\beta}; \quad Pr_{eq} = \text{const} \tag{11}$$

For air at reentry flight conditions, values of $\alpha = 0.4$, $\beta = 0.8$ and $Pr_{eq} = 0.75$ were chosen using the results of Viegas and Howe.[2] The solutions were obtained by a finite-difference method as part of an investigation of radiating boundary layers,[3] and the results are shown in Fig. 1b. While the $(c_f)_e$ and st_e parameters continue to decrease monotonically with M_e, the $(c_f)_r$ and st_r parameters approach asymptotic limits with an error of less than 4% for $M_e > 10$. Using the parameter definitions, Eq. (11), and the hypersonic approximation $\tilde{h}_e = [(\gamma_e - 1)M_e^2/2]^{-1}$ gives the relationship

$$\frac{(c_f)_e Re_e^{1/2}}{(c_f)_r Re_r^{1/2}} = \frac{st_e Re_e^{1/2}}{st_r Re_r^{1/2}} = \left(\frac{\gamma_e - 1}{2}\right)^{-(\beta-\alpha)/2} M_e^{-(\beta-\alpha)} \tag{12}$$

Substitution in Eq. (12) of the values of α and β used previously, $\gamma_e = 1.4$, and the asymptotic values of $(c_f)_r Re_r^{1/2}$ and $st_r Re_r^{1/2}$ gives

$$\begin{aligned} (cf)_e Re_e^{1/2}/(2n+1)^{1/2} &= 1.28\, M_e^{-0.4} \\ st_e Re_e^{1/2}/(2n+1)^{1/2} &= 0.69\, M_e^{-0.4} \end{aligned} \tag{13}$$

as convenient, approximate formulas.‡

A comparison of these results with boundary layer solutions using the exact gas properties of Viegas and Howe[2] obtained by Dennar[3] and by Chapman[4] are shown in Fig. 2 where Chapman's results for the $(c_f)_e$ and st_e parameters have been converted to $(c_f)_r$ and st_r parameters. The agreement is good except for Chapman's results at $p_e = 0.1$ atm. When a check was performed on Chapman's calculations for this condition, the results for $(c_f)_r$ and st_r were within 6% and 2% respectively of the hypersonic limit values. The discrepancy between the two calculations is attributed to the different methods used to integrate the boundary-layer equations.

† When the hypersonic approximations $h_\infty = 0$ and $u_e = u_\infty \cos\theta$ are used, $\tilde{h}_e = \tan^2\theta$ where θ is the wedge or cone semi-vertex angle.

‡ Although the form of Eq. (13) is the same as that given by the "reference temperature" method, the numerical values are different.

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Determination of the Stiffness of an Elastic Bar

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Introduction

IN Ref. 1 a method for nondestructive determination of the buckling load was proposed. The spring constants, which represent the actual boundary conditions, were obtained by measurements of the end deflections and rotations of the bar caused by a lateral load. The boundary conditions so obtained were inserted in the stability equations in order to predict buckling.

A different approach to the nondestructive testing method for the determination of the buckling load is presented in Refs. 2–5. The method is based on the observation that buckling loads of columns are raised if end restraint stiffnesses are increased while the flexibility of the member is decreased. The efforts are directed toward the establishment of empirical correspondence rules relating critical loads. As stated there the results obtained in this way are encouraging but much additional work remains to be done.

Although the method proposed in Refs. 2–5 is still impractical it poses a very attractive feature. A knowledge of

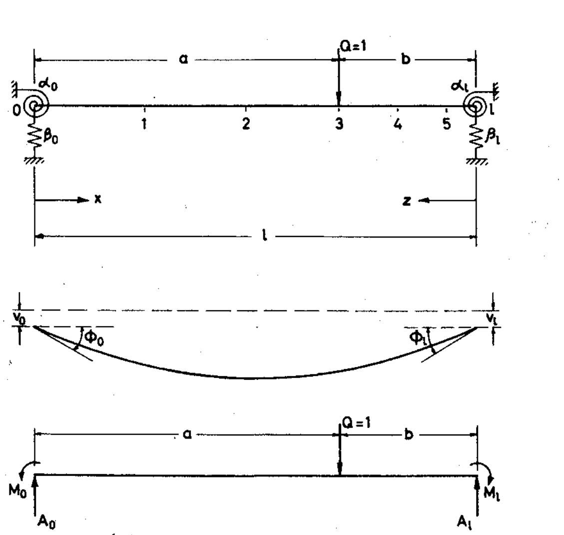

Fig. 1 Spring constrained beam loaded by a lateral force $Q = 1$.

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